Assignment 0:
Rule Induction

15-312: Principles of Programming Languages

Out: Tuesday, January 15th, 2013
Due: Tuesday, January 22nd, 2013, 11:59PM

1 Course Mechanics

Task 1.1 (4 pts).

1. Dolores and Toby are discussing Problem 3 by IM. Meanwhile, Toby is writing up his solution to that problem.
2. Amy, Jeff, and Chris split a pizza while talking about their homework, and by the end of lunch, their pizza box is covered with notes and solutions. Chris throws out the pizza box and the three go to class.
3. Ian and Jeremy write out a solution to Problem 4 on a whiteboard in Newell-Simon Hall. Then, they erase the whiteboard and run to the atrium. Sitting at separate tables, each student types up the solution on his laptop.
4. Nitin and Margaret are working on this homework over lunch; they write out a solution to Problem 2 on a napkin. After lunch, Nitin pockets the napkin, heads home, and writes up his solution.

2 Shuffling cards

Task 2.1 (10 pts). Prove the following (by giving a derivation). There are at least two ways to do so.

$$\mathsf{unshuffle}(\mathsf{cons}(\heartsuit, \mathsf{cons}(\spadesuit, \mathsf{cons}(\spadesuit, \mathsf{cons}(\diamondsuit, \mathsf{nil})))),\quad \mathsf{cons}(\spadesuit, \mathsf{cons}(\diamondsuit, \mathsf{nil})),\quad \mathsf{cons}(\heartsuit, \mathsf{cons}(\spadesuit, \mathsf{nil})))$$

Task 2.2 (5 pts). What was the other way? (describe briefly, or just give the other derivation)

Task 2.3 (15 pts). Prove that unshuffle has mode $(\forall, \exists, \exists)$. That is, prove the following:

For all s_1, if s_1 stack, then there exists s_2 and s_3 such that $\mathsf{unshuffle}(s_1, s_2, s_3)$.

Task 2.4 (15 pts). Give an inductive definition of separate, a judgment similar to unshuffle that relates a stack of cards to two "un-shuffled" sub stacks where all of the red cards (suits ♢ and ♡) are in one stack and all the black cards (suits ♣ and ♠) are in the other.

Task 2.5 (5 pts). Hopefully, your definition of separate will have not just the mode $(\forall, \exists, \exists)$, but the stronger mode $(\forall, \exists!, \exists!)$. What does this mode mean? Why does unshuffle *not* have this mode?

3 Cutting cards

Task 3.1 (6 pts). What is the induction principle for these judgments? You may want to examine the induction principle for even and odd natural numbers from PFPL.

Task 3.2 (15 pts). Prove well-formedness for the even judgment. That is, prove "For all s, if s even then s stack."

Task 3.3 (10 pts). Prove the following theorem:

> For all S, if
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> 1. $S(\mathsf{nil})$.
> 2. For all c_1, c_2, and s, if c_1 card, c_2 card, and $S(s)$, then $S(\mathsf{cons}(c_1, \mathsf{cons}(c_2, s)))$.
>
> then for all s, if s even then $S(s)$.

Task 3.4 (15 pts).
Using the derived induction principle from the previous task (you can use the induction principle from the previous task even if you do not do the previous task!), prove the following:

> For all s_1, s_2, s_3, if s_2 even, s_3 even, and $\mathsf{cut}(s_1, s_2, s_3)$, then s_1 even.